

27th International Congress on Waterborne Transportation, Shipbuilding and Offshore Constructions

Rio de Janeiro/RJ, 23rd to 25th October 2018

A Vessel Weather Routing Scheduler

Rodrigo Uchôa Simões, UFRJ, Rio de Janeiro/Brazil, rodrigo.usimoes@poli.ufrj.br

Antonio Augusto Martins Teixeira de Freitas, UFRJ, Rio de Janeiro/Brazil, antonioamtfreitas@poli.ufrj.br

Crístofer Hood Marques, FURG, Rio Grande do Sul/Brazil, cristerhood@furg.br

Jean-David Caprace, UFRJ/COPPE, Rio de Janeiro/Brazil, jdcaprace@oceanica.ufrj.br

Abstract

Voyage optimization in maritime transportation is essential to ensure good conditions of the operation – such comfort of the crew, reduce time of voyage and to maintain the efficiency and sustainability in the way to ensure cost competitiveness of ship operations. The constant need to maximize profit motivates even more developments and improvements in time organization and travel optimization systems. In fact, these technologies can lead to avoiding heavy sea conditions and to a significant reduction in travel time, fuel consumption and Green House Gas emissions. This work presents travel optimization schedules based on the Dijkstra algorithm and on Kwon's added resistance modelling. Supported in the vessel's movement and stochastic weather conditions, the forecast model evaluates the voyage according to different objectives: to reach the port of destination with the minimum travel time, to spend the least amount of fuel, to travel in the lowest conditions of Beaufort – in the best sea conditions - and to travel with the minimum head sea – the condition which the waves find the ship with less angulation relative to its surge movement. Therefore, the optimal route is obtained by considering both voluntary and involuntary sailing speed reduction. By utilizing a decision support tool, the ship's crew may select the optimum route according to their objectives. The validation of the model is performed by comparing the results obtained with established commercial routes. It is then possible to compare, by the means of knowing the vessel's and main engine characteristics, the amount of fuel consumed on the travel time for the determined routes. Work that opens the way for the consideration of greenhouse emissions.

1. Introduction

The continuous growth of the world population and of its standard of living, together with depletion of local resources, increases the dependency of the world economy on international trade. Moreover, several trends with relevant implications for maritime transport continue to gradually unfold and raise attention, such as the rapid expansion of electronic commerce (e-commerce) and growing concentration in the liner shipping market. Maritime transport accounts for the biggest amount of world trade and the projections for medium-term is to continue expanding, with volumes growing at an estimated compound

annual growth rate of 3.2% between 2017 and 2022. Volumes are set to expand across all segments, with containerized and major dry bulk commodities trade recording the fastest growth. In that way, the need to reduce fuel consumption and Green House Gas emissions (GHG are gases that trap heat in the atmosphere and has effects on climate changing) continues to encourage research. It is known that ship routing is a procedure to determine an optimal route for ocean voyages based on weather forecasts, characteristics of a particular ship and sea conditions. The principles of ship performance analysis, considering the problem of a routing

algorithm for optimizing trajectory and the advance in marine weather forecasting, has allowed the navigation industry to reduce voyage time, fuel consumption, cargo and hull damages and GHS, which is an actual known problem in maritime transportation.

This optimization problem has received attention of many researchers in past years, *Bijlsma* (1975) aims to minimize or maximize functions, expressed as integrals, in order to find extremals. The optimization is achieved through variation of the parameters that control the trajectory, for example time or velocity. The isochrone method proposed by *James* (1987) is a practical deterministic method for calculating the minimum time route. The optimization is determined by varying ship headings while assuming constant engine power. The isopone method of *Spaans* (1995), finds the optimal track by defining planes of equal fuel consumption (energy fronts) instead of time fronts. In addition to above mentioned algorithms, recently there have been significant advances in the modified isochrone method, *Lin et al.* (2013) applied a 3D model that considers the variation of sea conditions during voyage and avoids land. There is also the augmented Lagrange multiplier of *Tsujimoto* (2006), the genetic algorithm of *Becker* (2006) and the *Dijkstra* algorithm of *Takashima* (2009) that is used for this paper. The ship speed loss due to wind and waves of *Kwon* (2008).

It is important to realize that the optimum route cannot be found by evaluating separately the aspects involved. Normally, the voyage optimization has multiple, often conflicting, objectives, such as: minimizing costs regardless of arrival time, punctual time of arrival, safety, and passenger comfort. In most cases, improving one objective may reduce efficiency of another. Each attribute, therefore, requires a weighting of importance. Some business models shipping companies prioritise on-time arrival and shorter transit times over reduced fuel consumption. For other companies, providing a 'green service' has a higher priority.

This paper describes a weather routing scheduler for determining optimal ship routes, based on *Dijkstra's* algorithm. In the routing algorithm are established weight functions based on different objectives to achieve. All possible routes are considered and the algorithm must find the one of minimum weight, obtaining the optimal track. The

weights approached in this paper are voyage time, wave encounter angle, wave height and the specific fuel oil consumption (SFOC).

When attempting to find the best route for a marine transport, it is important to consider the main obstacles of the voyage. The main concern of this paper is the weather conditions, and its aspects that will be focused on are wave direction and specific size. Therefore, the reliability of the optimal route derived from the ship weather routing scheduler is based on the following parameters: the accuracy of weather forecasted data and the estimated ship behaviour in such ocean wave conditions.

2. Methodology

In this section it is presented the methodology describing the processes for weather forecasting data mining, calculation of added resistance due to sea conditions, voluntary speed reduction and optimization of the route.

2.1. Weather forecasting

In order to calculate the best route, forecasting the weather is of extreme importance. To do so, collected data from previous years is to be gathered and organized on a $1^\circ \times 1^\circ$ grid created by latitude and longitude lines, so that each part of the ocean has a predicted weather characteristic. The data was obtained from The European Centre for Medium-Range Weather database.

2.2. Added Resistance and Speed Reduction

According to *Sen and Padhy* (2015), the resistance in still water (R_{SW}) is the only component in calm water total resistance (R_T). In open sea sailing, the total additional resistance (R_{ADD}) must consider the additional resistance due to waves (R_{AW}), wind (R_W) and currents (R_C) such as given in Eq. 1.

$$R_{ADD} = R_{AW} + R_W + R_C \quad (1)$$

So, the total resistance (R_T) is given by the sum of still water resistance (R_{SW}) and additional resistance (R_{ADD}), as shown in equation:

$$R_T = R_{SW} + R_{ADD} \quad (2)$$

The additional resistance can be represented either by an increase in the required power to

maintain the speed, or by a speed reduction for a given constant power. In order to simplify the calculations, only the wave additional resistance parcel will be considered. The additional resistance is represented by a speed reduction, and it is considered both voluntary and involuntary speed reduction.

2.2.1. Involuntary speed reduction

The involuntary speed loss is not intentional and it is due to an increase in resistance as result of the presence of waves.

Beaufort Number

Nowadays, the scale is used to set the sailing condition based on wind speed and wave height ranges throughout the oceans (*Beaufort*, 1805). In the marine transport analysis, the scale is used to verify each part of the ocean's sailing conditions. In this paper, it is used in the calculating method in order to model the weights functions. The Beaufort Number is used directly on best sea conditions weight function. And still is used to calculate added resistance.

Kwon Method

In order to estimate the added resistance due to weather conditions on a vessel, the chosen method was developed by *Kwon* (2008). The method calculates the approximate involuntary speed loss of a displacement type vessel, and its application is quite simple. It is a good introduction on this subject and is based on the ship's hull form, encounter angle and sea state.

The method relies on using the speed loss as a comparison between the nominal velocity of the ship (the velocity on still waters) with its actual velocity in varying weather conditions. The comparison is made through the following expressions:

$$\frac{\Delta V}{V_1} 100\% = C_\beta C_U C_{Form} \quad (3)$$

$$V_2 = V_1 - \left(\frac{\Delta V}{V_1} 100\% \right) \frac{1}{100\%} V_1 \quad (4)$$

$$V_2 = V_1 - C_\beta C_U C_{Form} \frac{1}{100\%} V_1 \quad (5)$$

Where;

V_1 : Nominal operating ship speed in still water, given in m/s;

V_2 : Actual operating ship speed, due to weather conditions, given in m/s;

ΔV : Ship's speed loss ($V_1 - V_2$), given in m/s;

C_β : Direction reduction coefficient, dependent on the weather's direction angle (with respect to the ship's bow) and the BN; as shown in Table 1.

C_U : Speed reduction coefficient, dependent on the ship's block coefficient (C_B), as shown in Table 2.

C_{Form} : Ship form coefficient, as shown in Table 3.

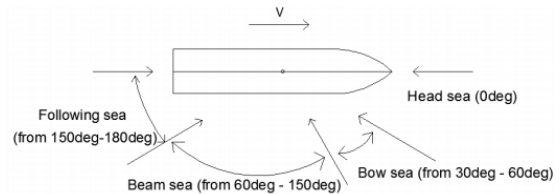


Figure 1 – Wave Encounter Category Angles.

Table 1 – Direction reduction coefficient C_β .

Encounter angle [deg]	C_β
0 – 30	$2C_\beta = 2$
30 – 60	$2C_\beta = 1.7 - 0.03[(BN - 4)^2]$
60 – 150	$2C_\beta = 0.9 - 0.06[(BN - 6)^2]$
150 – 180	$2C_\beta = 0.4 - 0.03[(BN - 8)^2]$

Table 2 – Speed reduction coefficient C_U .

C_B	Ship loading conditions	C_U
0.55	normal	$1.7 - 1.4Fn - 7.4Fn^2$
0.6	normal	$2.2 - 2.5Fn - 9.7Fn^2$
0.65	normal	$2.6 - 3.7Fn - 11.6Fn^2$
0.7	normal	$3.1 - 5.3Fn - 12.4Fn^2$
0.75	loaded or normal	$2.4 - 10.6Fn - 9.5Fn^2$
0.8	loaded or normal	$2.6 - 13.1Fn - 15.1Fn^2$
0.85	loaded or normal	$3.1 - 18.7Fn + 28.0Fn^2$
0.75	ballast	$2.6 - 12.5Fn - 13.5Fn^2$
0.8	ballast	$3.0 - 16.3Fn - 21.6Fn^2$
0.85	ballast	$3.4 - 20.9Fn + 31.8Fn^2$

Table 3 – Ship form coefficient C_{Form} .

Type of (displ.) ship	C_{Form}
All ships (except container ships) in loaded loading condition	$0.5BN + BN^{6.5}/(2.7 \Delta^{2/3})$
All ships (except container ships) in ballast loading	$0.7BN + BN^{6.5}/(2.7 \Delta^{2/3})$

condition Container ships in normal loading conditions	$0.7BN + BN^{6.5}/(22 \Delta^{2/3})$
---	--------------------------------------

2.2.2. Prediction on Calm Water Resistance

For this paper, the prediction of Still Water Resistance is done by the Holtrop and Mennen' (1982) method. So, in every point that is verified an effect of sea in ship speed, it then necessary to recalculate the of the ship. This is done in order to predict the influence on main engine power demand considering the speed loss due to waves in the sea.

2.2.3. Voluntary speed reduction

For this paper, the voluntary speed reduction applies to avoid such slamming conditions (ABS, 2011). Therefore, in order to calculate the voluntary speed reduction, it is considered a standard speed profile formulated by *American Bureau of Shipping* (ABS, 2011). This profile is applied based on the significant wave height and relates the speed reduction with the occurrence of slamming, as shown in Table 4.

Table 4 – Standard speed profile for slamming local prediction.

Significant wave height [m]	Speed reduction
$0 < H_s \leq 6$	0
$6 < H_s \leq 9$	25% V_1
$9 < H_s \leq 12$	50% V_1
$12 < H_s$	75% V_1

2.3. Route optimization

2.3.1. Dijkstra's algorithm

Dijkstra's algorithm is used for finding the shortest paths between nodes in a graph. The algorithm exists in many variants. This paper relies to algorithm presented by *Takashima* (2009). The original variant finds the shortest paths between two nodes, but a more common variant fixes a single node as the source and finds de optimal track from the source to all the other nodes in the graph, producing a shortest-path tree among all the possible routes in the graph.

According to *Fan and Shi* (2010), among the available algorithms this one is the most classical

and mature for obtaining the optimal route. With the advent of computing power and suitable digitalization of the open sea area, this algorithm is practical for the problem and easy to implement. In this paper, it is used for finding the optimal routes between the origin node and a single destination node.

As *Zhu et al.* (2016) has defined, it is considered a direct matrices $G = (V, E)$, with n vertexes and E real valued weights sides. Where V is a collection of initial vertexes, E is a set of real valued weights sides and S is a set of vertexes which have found the minimum time travel route from the starting vertex to themselves. In the diagraph the track from the vertex v_0 to the vertex v_n is a sequence of vertexes $v_0, v_1, v_2, \dots, v_n$. $V-S$ is a set of vertexes that have not found the optimum route between the origin and themselves. The steps for Dijkstra's algorithm are:

- 1) Use the weighted adjacency matrix arcs to represent the directed graph and arcs(s,i) is the weight from the node s to i . Suppose S equals $\{V_S\}$ and V_S is the origin point. Suppose $dist[i]$ equals the minimum time track from node V_S to node V_i .

$$dist[i] = \begin{cases} 0 & i = s; \\ arcs(s, i) & i \neq s, < V_S, V_i > \in E; \\ \infty & i \neq s, < V_S, V_i > \notin E; \end{cases} \quad (6)$$

- 2) V_j is the end of the next optimum path. Select the node V_j by:

$$dist[j] = \min\{dist[k] \mid V_k \in V - S\}, S = S \cup \{V_j\} \quad (7)$$

- 3) If $dist[j] + arcs(j, k) < dist[k]$, then: $dist[k] = dist[j] + arcs(j, k), (\forall V_k \in V - S)$ (8)

- 4) Repeat step 2 and 3 until $S = V$.

In a scheme formulated below, this algorithm can be illustrated. The scheme given in Figure 2 and Table 5 illustrates this algorithm and consists in a graph of vertexes s, u, x and v . In this example, the aim is to find the optimum track, by searching the minimum total weight path starting from the vertex s and achieving the node v . Firstly, a temporary weight value should be assigned to each node of the mesh. This is done to identify the origin. So, to identify the origin, it temporary weight is set zero. For the other nodes, the weight is temporary set an infinity number.

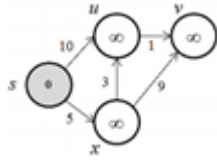


Figure 2 - Illustration of Dijkstra's time zero step example.

Table 5 – Dijkstra's time zero step.

Vertex	Visited?	Distance	Predecessor
s	Yes	0	-
u	-	-	-
v	-	-	-
x	-	-	-

Then, it is necessary to define the initial node as the current node and mark the remaining nodes as unvisited, creating a set of unvisited nodes. Next, see Figure 3 and Table 6, the adjacent vertexes, in this case *u* and *x*, have not been visited yet. The node *s* is then at a 10 unit of weight in relation to the start node *u* and at a 5 unit of weight in relation to *x*. So, it is registered the predecessor of these adjacent vertexes and its paths weighs.

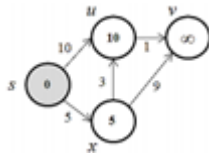


Figure 3 - Illustration of Dijkstra's first step example.

Table 6 – Dijkstra's first step.

Vertex	Visited?	Distance	Predecessor
s	Yes	0	-
u	No	10	s
v	No	-	-
x	No	5	s

The node that has the minimum time value in relation to *s* is the node *x*, so the second vertex to visit is the *x*. In this step, since the weight of going from *x* to *u* is 3, then, the total weight of traveling from *s* to *u* by midpoint *x* would be equal to 8. So, for vertex *u*, it is adopted a new temporary weight equal to 8, see Figure 4 and Table 7, since the new value is smaller than previous 10 (path *s-u*). In other words, at each step that the algorithm gives, the provisional weight of the unvisited nodes is changed if the new temporary weight (alternative paths) is less than the previous weight. Note that as the path progresses, the predecessors are also

being registered, see Figure 3 and Table 6, as there is a need to store the predecessor in order to find the final path.

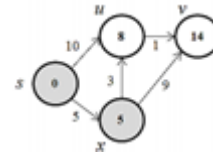


Figure 4 - Illustration of Dijkstra's second step example.

Table 7 – Dijkstra's second step.

Vertex	Visited?	Distance	Predecessor
s	Yes	0	-
u	No	8	x
v	No	14	x
x	Yes	5	s

The third step, since the weight of going from *x* to *u* (weight's 3) is less than to *v* (weight's 9), the next node to visit is *u*, see Figure 5 and Table 8. The final step is a path between node *u* and final node *v*. Since its weights 1, and the previous path weights (total weight of achieving node *u*) 8, the total voyage weight would be 9. Then the algorithm has found the track: *s-x-u-v*, which has the minimum total weight.

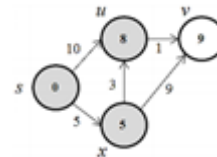


Figure 5 - Illustration of Dijkstra's third step example.

Table 8 – Dijkstra's third step.

Vertex	Visited?	Distance	Predecessor
s	Yes	0	-
u	Yes	8	x
v	No	9	u
x	Yes	5	s

2.3.2. Weight function

The weight function is the objective function that has to be optimized, since the faster track needs to be found. In this paper, as defined, the weight functions are voyage time between knots, encounter angle of waves, Beaufort number and fuel consumption.

2.3.3. Implementation

A Visual Basic for Application (VBA) script was made to solve the voyage optimization problem. The steps which code was built are presented in Figure 6.

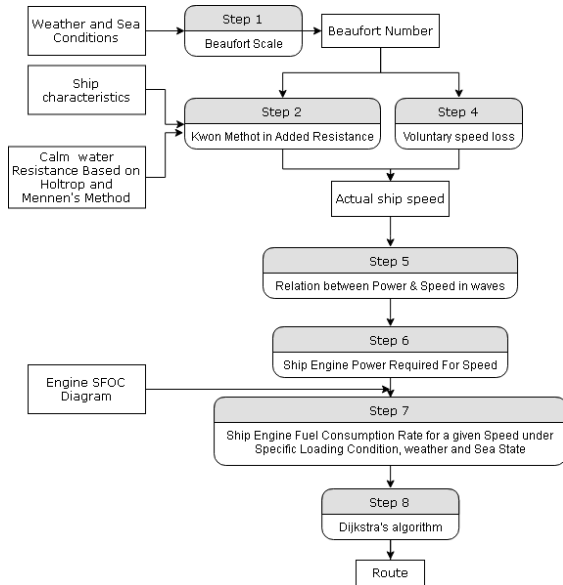


Figure 6 - Flowchart of the VBA code.

The input parameters in the code are: Ship Particulars, see Table 9, Latitude/Longitude from arrive port, Latitude/Longitude from destination port, number of days of delivered data and Load Condition. The load condition can be “loaded” or “ballast”, although, in this article, only loaded condition is approached.

In the minimum time, minimum Beaufort, minimum head sea and minimum distance routes, the engine power has been taken as a constant and the main parameters, which vary from point to point, were velocity, Beaufort Number, encounter angle and distance travelled, respectively. In the minimum fuel consumption voyage, the velocity has been taken as a constant and engine power, relative to consumption, vary from point to point.

This was done to analyse only the effects of the sea conditions on the voyages proposed. If this was not adopted, the algorithm would try to set maximum engine power, in order to reach the minimum voyage time. In the minimum fuel consumption route, the algorithm would try to set the optimum engine power constantly in the hole trip. So, in these cases, the effects of the sea conditions could not be analysed.

To simplify the search for the solution and reduce the computational time, the bow’s angle of the ship was taken constant between waypoints. In the seek to refine the solution, the user can determine as many waypoints. This can be done directly in the data presented, as desired. Note that the computation time increases exponentially in the same direction of increasing waypoints number.

3. Results and Discussions

In order to check the capability of the proposed method, a simulation was carried out. For the simulation, the departure point was the port of Tubarão, Espírito Santo in Brazil (-20.29°, 40.00°) and the destination was the port of Zhanjiang, Guangdong province in China (21.22°, 110.42°). The waves height and mean direction were from July 2018. During the voyage, two waypoints were considered: South Africa (-35.15°, 19.73°) and Singapore (1.29°, 103.68°). It is important to notice that waypoints are introduced only to give the general direction of the ship – and to reduce computation time - and assess correctly the angle between the true heading and the global swell. However, the ship was not constraint to pass by these waypoints. The selected ship for the study was an Ultra Large Ore Carrier (ULOC) class Valemax, which the principal particulars are shown in Table 10.

Table 9 – ULOC Main Particulars.

Particular	Value
Length [m]	360
Length on WL [m]	352.790
Breadth [m]	65
Draft [m]	23
Depth [m]	30
Sailing Speed [kn]	14
Volume [m ³]	454186
CB	0.85

The results of the optimized route combined with the total significant height of swell at various moment of the journey are shown in Figures 7 to 12. The dark green path is the route that take less days to achieve China; and, the red path is the minimum fuel consumption route.

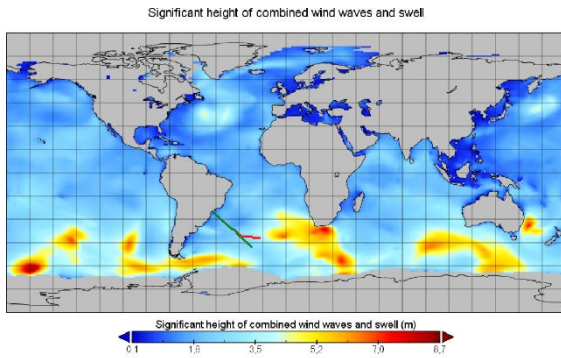


Figure 7 – Routes between day 1 and 5.
Significant height of combined wind waves and swell

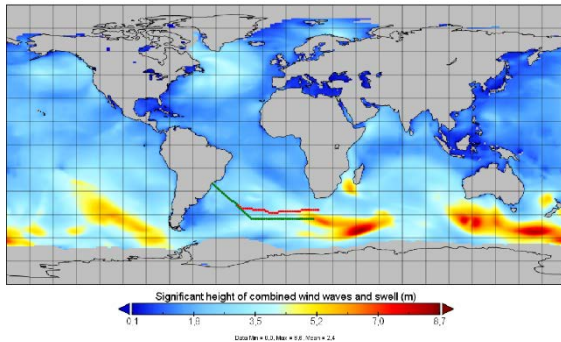


Figure 8 – Routes between day 6 and 10.
Significant height of combined wind waves and swell

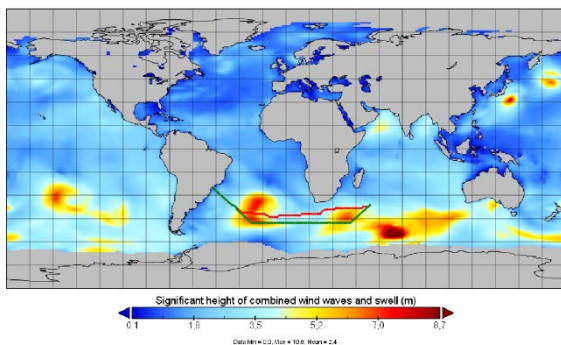


Figure 9 – Routes between day 11 and 15.
Significant height of combined wind waves and swell

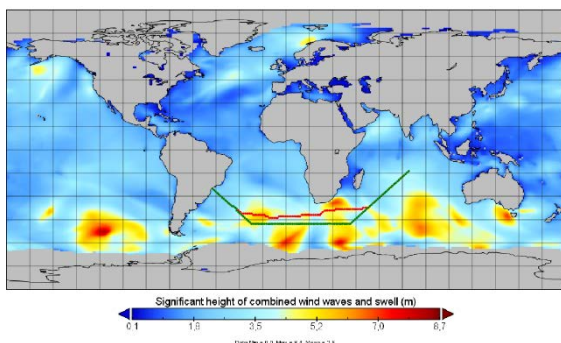


Figure 10 – Routes between day 16 and 20.

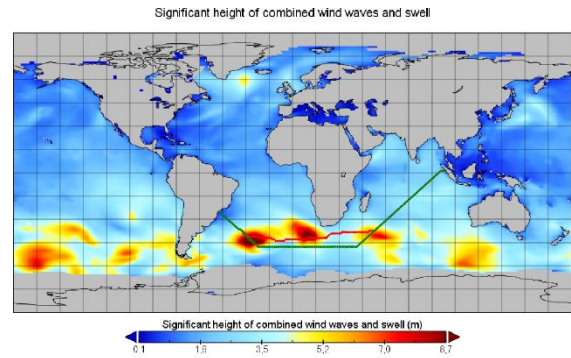


Figure 11 – Routes between day 21 and 25.
Significant height of combined wind waves and swell

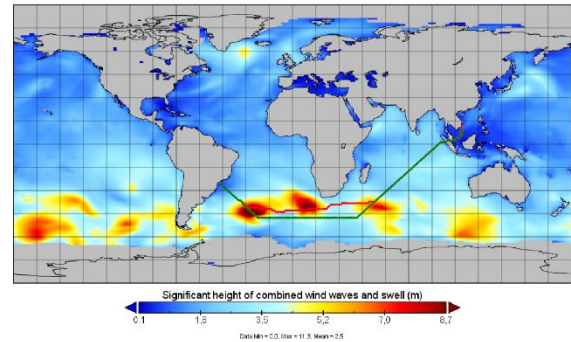


Figure 12 – Routes between day 26 and 29.

In the minimum time voyage, it can be observed that in the Atlantic Ocean, the ship has been in front of rough weather. For this reason, the ship headed a little south then in the minimum fuel consumption route. Besides, when the ship arrived at the Cap of Good hope, in the red path, it has been forced to slightly go north to avoid rude weather and therefore reduce the steaming total fuel consumption. Once the ship passed the south of Madagascar the algorithm chooses to go even more North for the same reasons.

travelled without bad weather conditions, which forced them to be equal in most part of Indian ocean. Once arrived in Indonesia it may be observed that weather is not any more affecting the route. Due to a problem of weather data low resolution ($1^{\circ} \times 1^{\circ}$ grid), the ship passed through Thailand instead of taking the Malacca channel.

It can be solved simply by using a better resolution of the weather data; however, the computation time is drastically penalized. Finally, the ship arrived in China after 29.70 days (712.80 hours) of navigation – minimum time route. For the proposed voyage, the algorithm calculated a total fuel consumption, in the minimum consumption route, about 1850 tons of crude oil after 28.75 days (690 hours) of navigation. In comparison, in

minimum time path, the ship spent 2360 tons of oil.

In the minimum Beaufort and head sea voyages, which can be seen in Figure 13, the ship takes paths very different from the other routes just looking after good conditions and less bow and beam waves, considering the comfort on-board. It can be said that in these routes, the voluntary reduction of speed was minimum, since good sea conditions were ahead of ship.

The pink path is the minimum Beaufort route; and, the brown path is the minimum head sea condition route.

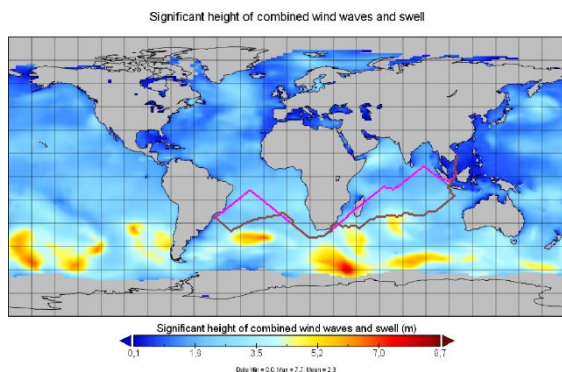


Figure 13 – Minimum Beaufort and Head Sea routes.

To illustrate the differences between the proposed routes, they can be seen in Figure 14. The total time, consumption and distance travel of each route can be seen in Table 10.

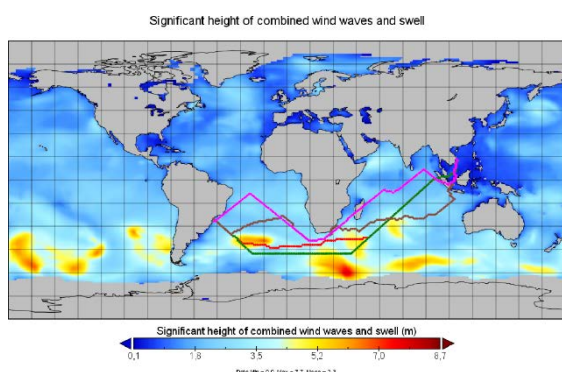


Figure 14 – Proposed Voyages.

Table 10 – Comparison Table of Results.

Route Objective	Time [days]	Distance [km]	Fuel [t]
Time	29.70	18747	2360
Beaufort	37.12	23338	2949
Head Sea	38.10	23744	3026
Fuel	28.75	19163	1850

4. Conclusions

In this paper, the results show that a reduction in the fuel consumption and voyage time can be done by choosing the proper path. In the proposed case, since the ship has not faced storms, the decision of looking forward the optimum engine power would be the best relative to both time and fuel consumption. It is important to notice that in the minimum fuel consumption voyage, the ship engine was forced to look for changes in power output to maintain the ship traveling with service speed, which is impractical in ranges such as from node to node (1° x 1° grid). For this purpose, this paper open spaces to studies about possibly of range that the engine can be vary power output during voyage.

Even though the research that presented basis for writing this paper did not consider all the countless variables that influence the route, it was notable that many optimum routes were found, each one with its purpose. Each route was notably different from the others, as its goals were completely unrelated.

For future analysis, the optimum cost-effective route may be looked into by pondering value to each of the found routes and combining them, as that is the usual main goal for any company, and the problem that engineers deal with. One of the limitations of this approach is that the involuntary speed reduction regards only the wave additional resistance parcel, so the influence of the wind and current should be considered in future improvements.

5. References

ABS (2011), *Guide for Slamming Loads and Strength Assessment for Vessels*, American Bureau of Shipping, Houston

BEAUFORT, F. (1805), *Beaufort Wind Scale*, U.K. Royal Navy

BECKER, R.; GLORIE, K. (2006). *A genetic algorithm for kidney transplantation matching*, 2006.

BIJLSMA, S.J. (1975), *On minimal-time ship routing*, Delft University of Technology

FAN, D.; SHI, P. (2010), *Improvement of Dijkstra's algorithm and its application in route planning*, 7 th Int. Conf. Fuzzy Systems and Knowledge Discovery (FSKD), Vol. 4, pp.1901-1904

HOLTROP, J., Mennen, G.G.J., 1982. *An approximate power prediction method*, Int. Shipbuilding. Prog. 29, 166–171.

JAMES, R.W. (1987), *Application of wave forecast to marine navigation*, US Navy Hydrographic Office, Washington

KWON, Y.J., 2008. *Speed loss due to added resistance in wind and waves* 3, 14–16 Nav. Archit. 3, 14–16.

LIN, Y.H.; FANG, M.C.; YEUNG, R.W. (2013), *The optimization of ship weather routing algorithm based on the composite influence of multi-dynamic elements*. Appl. Ocean Res. 43, pp.184-94

SEN, D.; PADHY, C.P. (2015), *An approach for development of a ship routing algorithm for application in the North Indian Ocean region*, Appl. Ocean Res. 50, pp.173-191

SPAANS, J.A. (1995), *New developments in ship weather routing*, Navigation Vol. 169, pp. 95-106

TAKASHIMA, K; MEZAOUI, B; SHOJI, R. (2009). *On the fuel saving operation for coastal merchant ships using weather routing*. International Journal on Marine Navigation and Safety of Sea Transportation.3.401-406.
0.1201/9780203869345.ch75.

TSUJIMOTO, M; TANIZWA, K. (2006). *Development of a Weather Adaptive Navigation System Considering Ship Performance in Actual Seas*. 2006. 10.1115/OMAE2006-92376.

ZHU, X.; WANG, H.; SHEN, Z.; HONGJUN L.V. (2016), *Ship weather routing based on modified Dijkstra algorithm*, 6th Int. Conf. Machinery, Materials, Environment, Biotechnology and Computer (MMEBC 2016), pp.696-697